

Micropillar resonator in a magnetic field: Zero and Finite temperature cases

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Abstract

In this work, we present a theoretical study of a quantum dot-microcavity system which includes a constant magnetic field in the growth direction of the micropillar. First, we study the zero temperature case by means of a selfconsistent procedure with a trial function composed by a coherent photon field and a BCS function for the electron-hole pairs. The dependence of the ground-state energy on the magnetic field and the number of polaritons is found. We show that the magnetic field can be used as a control parameter of the photon number, and we make explicit the scaling of the total energy with the number of polaritons. Next, we study this problem at finite temperatures and obtain the scaling of the critical temperature with the number of polaritons.

Key words: quantum dot, microcavity, polaritons, B.E.C

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1 Introduction

In the last years, the study of many microsystems that confine light and provide the interaction with an active medium has made possible the observation of interesting phenomena such as, for example, the control of the spontaneous emission (Purcell effect)[1,2,3], or the enhancement of ground state occupation for exciton-polaritons at low temperatures [4,5] (in the search for BEC of polaritons). Many interesting applications to quantum computation [6] and other areas are envisaged.

An exciton-polariton is a quasi-particle composed by an exciton and a photon in the strong coupling regime. These quasi-particles have less reduced effective

mass than atoms or excitons and, consequently, a relatively high condensation temperature is expected.

In this work, we study the effects of an homogeneous magnetic field, applied in the growth direction of a quantum dot-micropillar system on the energy, the mean number of photons in the microcavity, and the critical temperature for BE condensation of polaritons. Our calculations assume a conserved number of polaritons. Due to the fact that polaritons indeed decay, such an assumption means that their lifetime is much longer than the time required to achieve thermal equilibrium at a fixed number of polaritons [7].

The paper has been written as follows: In section II, we describe the system and the theoretical model to be used below, in section III, we describe the selfconsistent method used to obtain the ground-state energy. In the next section, we explain the extension of the method to finite temperatures and, finally, in the last section we present numerical results for the ground state energy, mean number of photons and the critical temperature.

2 Theoretical Model

The heterostructure that we are interested in is a circular pillar grown by periodic deposition of layers of GaAs and AlGaAs (Bragg mirrors). In the center of this pillar a λ -cavity-defect of GaAs is placed. It contains in the middle a set of quantum dots of GaInAs. When the radius of this micropillar resonator is about $0.5\mu\text{m}$, one can assume that the microcavity operates with a single electromagnetic mode coupled to the quantum dot excitations (the next cavity mode is separated ~ 20 meV) [8]. Furthermore, we consider an homogeneous magnetic field along the z direction (the pillar growth direction) [9].

In our calculations, we take the lateral confinement in the quantum dot as a parabolic potential. A Landau basis of single particle states for electrons (holes) is used. The parameters have been taken as follows. Effective in-plane masses $m_e = 0.05 m_0$, $m_h = 0.07 m_0$, where m_0 is the free electron mass, are used. $\hbar\omega_0 = 1\text{meV}$ is the confinement energy. $\hbar\omega = 1$ meV is the photon energy, measured with respect to the nominal band gap. The photon-matter coupling strength is given by $g = 0.5$ meV. We take the Coulomb interaction constant $\beta = 2.73 \sqrt{B}$ meV, and the cyclotronic frequencies for electrons and holes as $\hbar\omega_c^{e(h)} = 1.15 * 10^{-1} B / m_{e(h)}$ meV, where B is given in Teslas. The Hamiltonian for this problem is

$$\hat{H} = \sum_n (E_n^{(e)} e_n^\dagger e_n + E_n^{(h)} h_n^\dagger h_n) + \sum_n (t_{kn}^{(e)} e_k^\dagger e_n + t_{k\bar{n}}^{(h)} h_{\bar{k}} h_{\bar{n}})$$

$$\begin{aligned}
& + \frac{\beta}{2} \sum_{r,s,u,v} \langle r, s | \frac{1}{r} | u, v \rangle e_r^\dagger e_s^\dagger e_v e_u + \frac{\beta}{2} \sum_{r,s,u,v} \langle r, s | \frac{1}{r} | u, v \rangle h_r^\dagger h_s^\dagger h_{\bar{v}} h_{\bar{u}} \\
& - \beta \sum_{r,s,u,v} \langle r, s | \frac{1}{r} | u, v \rangle e_r^\dagger h_s^\dagger h_{\bar{v}} e_u + \hbar\omega a^\dagger a + g \sum_n (a^\dagger e_n h_{\bar{n}} + a h_{\bar{n}}^\dagger e_n^\dagger) \quad (1)
\end{aligned}$$

3 $T = 0$

We use a BCS-variational method with a trial function given by

$$|u_n, v_n, \sigma\rangle = |\sigma\rangle \otimes \prod_n^{Nstates} (u_n + v_n e_n^\dagger h_n^\dagger) |0\rangle, \quad (2)$$

where $|\sigma\rangle$ is a coherent state for the photons, and $\prod_n^{Nstates} (u_n + v_n e_n^\dagger h_n^\dagger) |0\rangle$ describes the electron-hole pairs. The number of polaritons is defined as $\hat{N}_{pol} = \hat{N}_{pairs} + \hat{N}_{ph}$. By minimizing the $\langle \hat{H} \rangle$ with respect to σ and v_n we get the following equations

$$(\hbar\omega - 2\mu_{ex})\sigma - g \sum_i \frac{1}{2} \frac{\Delta_i}{\sqrt{\Delta_i^2 + (\epsilon_i - \mu_{ex})^2}} = 0, \quad (3)$$

$$\Delta_k = \beta \sum_{i, i \neq k} [i : k] \frac{1}{2} \frac{\Delta_i}{\sqrt{\Delta_i^2 + (\epsilon_i - \mu_{ex})^2}} + g\sigma, \quad (4)$$

the latter is a generalized gap equation. $[i : j] = \langle ij | \frac{1}{r} | ji \rangle$. The explicit expression for the mean number of polaritons is

$$N_{pol} = |\sigma|^2 + \frac{1}{2} \sum_i \left(1 - \frac{(\epsilon_i - \mu_{ex})}{\sqrt{\Delta_i^2 + (\epsilon_i - \mu_{ex})^2}} \right), \quad (5)$$

where we used the standard parametrization for the occupations:

$$v_k^2 = \frac{1}{2} \left(1 - \frac{(\epsilon_i - \mu_{ex})}{\sqrt{\Delta_i^2 + (\epsilon_i - \mu_{ex})^2}} \right), \quad (6)$$

μ_{ex} (chemical potential) is the Lagrange multiplier associated to the conservation of N_{pol} , and ϵ_i is the pair energy.

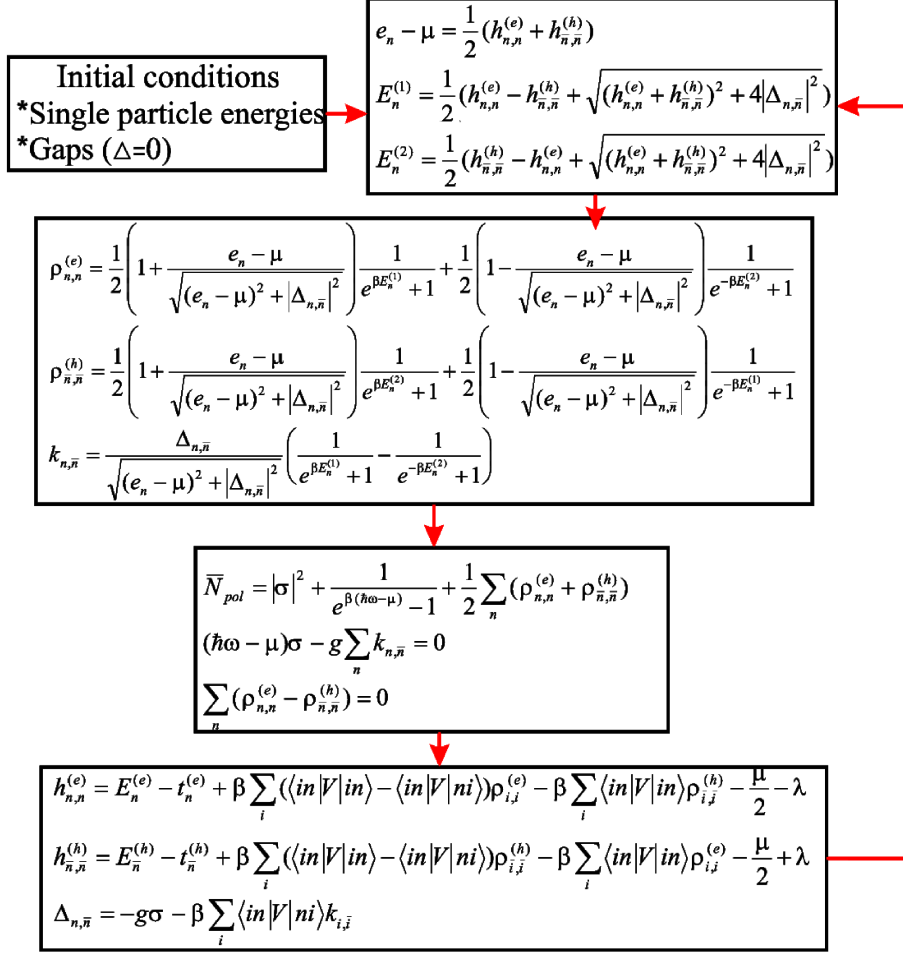


Fig. 1. Selfconsistent procedure at finite temperatures.

$$\begin{aligned}
 \epsilon_i &= \frac{1}{2}(E_i^{(e)} + E_i^{(h)} + t_{i,i}^{(e)} + t_{i,i}^{(h)}) - \frac{\beta}{2}[i : i] \\
 &\quad - \beta \sum_{j, (j \neq i)} [i : j] v_j^2,
 \end{aligned} \tag{7}$$

Eqs. (3-5) are solved iteratively to obtain σ , μ_{ex} and the gap parameters Δ_i .

4 $T \neq 0$

At finite temperatures, the starting point is the density matrix [10,11,12],

$$D = \frac{e^{-\beta K_{ph}}}{Z_{ph}} \frac{e^{-\beta K_{eh}}}{Z_{eh}} \tag{8}$$

Where $Z_{ph} = Tre^{-\beta K_{ph}}$, $Z_{eh} = Tre^{-\beta K_{eh}}$, $K_{ph} = \hbar\omega - \mu_{ex}$, and K_{eh} is expressed in terms of single- particle energies and gap functions. The thermodynamical potential

$$\phi(D) = Tr(DH) - \mu\bar{N}_{pol} + \frac{1}{\beta}Tr(D \ln D) \quad (9)$$

Should be minimized with respect to σ and the v_n . The selfconsistent procedure is schematically sketched in Fig.1.

5 NUMERICAL RESULTS

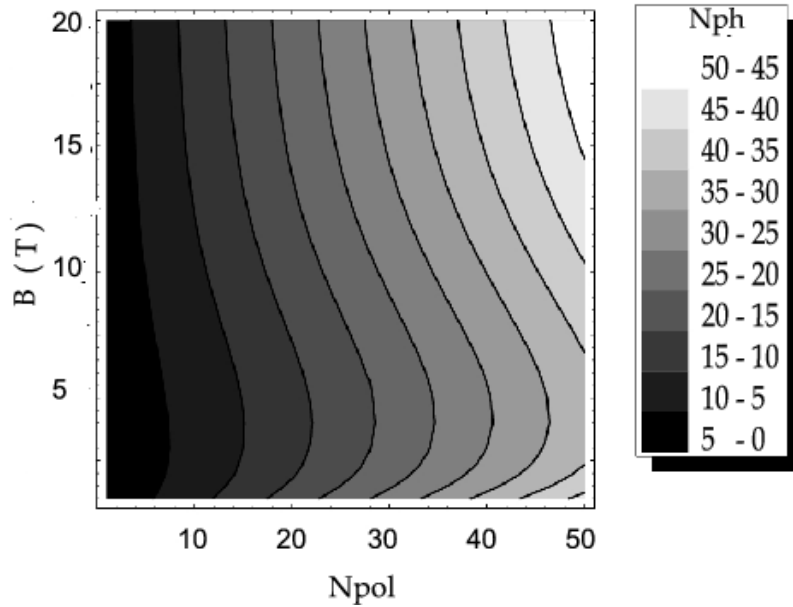


Fig. 2. The number of photons as a function of the number of polaritons and the magnetic field, $\hbar\omega = 1$ meV and $T=0$.

At $T = 0$, we show in Fig. 2 the mean number of photons in the cavity as a function of B and the number of polaritons. At fixed polariton number we obtain a non monotonous dependence on B , which can be understood in terms of the effect of B on the pair energies. At low B , the energies decrease because of a term of the form of $\sim \frac{\hbar e}{2}(\frac{1}{m_e} + \frac{1}{m_h})B\langle l_e \rangle$, where the mean value of the electron angular momentum becomes negative. Thus the detuning ($E_{photon} - E_{pair}$) increases and the mean number of photons decreases. On the other hand, for large values of B , the contribution of the lowest Landau level to the energy, $\frac{\hbar e}{2}(\frac{1}{m_e} + \frac{1}{m_h})B$, makes the detuning lower, thus augmenting the number of photons.

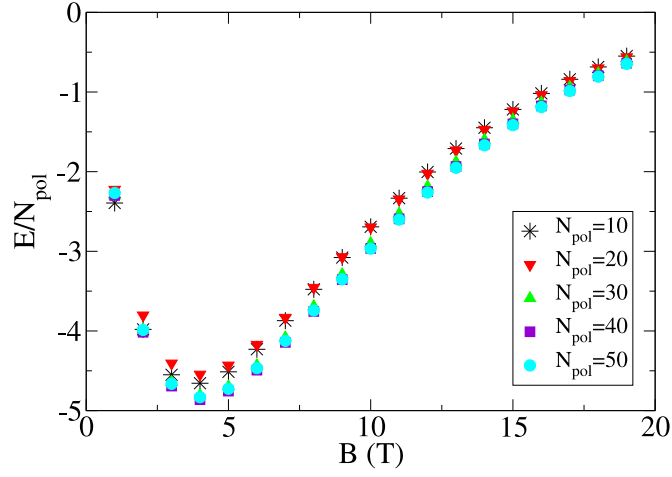


Fig. 3. Scaling of the total energy, $T = 0$ and $\hbar\omega = 1$ meV.

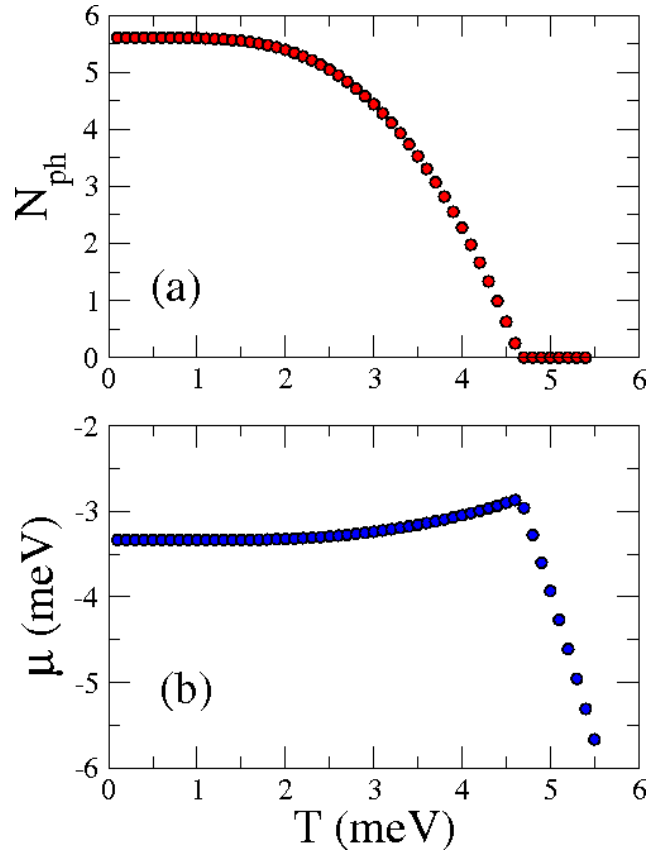


Fig. 4. The number of photons as a function of temperature for $\hbar\omega = 1$ meV, $\hbar\omega_0 = 1$ meV and $N_{pol} = 10$.

The total energy as a function of B is drawn in Fig. 3. The change in the slope from low B to large B has the same origin as discussed above. Coulomb

interactions at large B values gives a contribution $\sim -\sqrt{B}$ which in conjunction with the dominant term from the lowest Landau level makes the curve concave. The scaling $E \sim N_{pol}$ could be taken as an indication that the effective interactions between quasi-particles (polaritons) is very weak (we expect a contribution $\sim N_{pol}^2$ coming from interactions). The scaling is completely natural for high B , as it happens for excitons [13].

In Fig. 4. We show the calculated mean number of photons in the condensate ($|\sigma|^2$) and the chemical potential as a function of temperature for a 10-polariton system at $B = 7$ Teslas. The transition at $T \approx 4.6$ meV is interpreted as Bose condensation of polaritons in the same way that it happens for the Dicke model [14]. Notice that, as follows from our equations (see Fig. 1), a zero value for σ implies zero gap functions $\Delta_{n,\bar{n}}$.

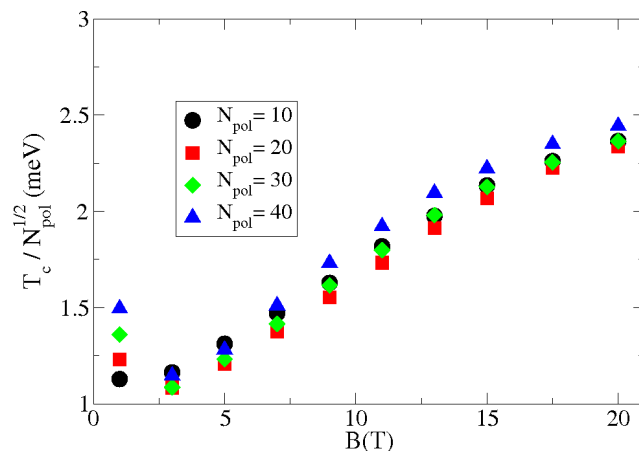


Fig. 5. Critical temperature vs magnetic field.

The results of more extensive calculations for different N_{pol} numbers and magnetic fields are represented in Fig.5. The dependence $T_c \sim N_{pol}^{1/2}$ is typical of free 2D bosons in an harmonic potential. Thus, the approximate scaling with $N_{pol}^{1/2}$ again reinforces the picture of almost noninteracting effective quasi-bosons (polaritons). The obtained values of T_c are of the order of the pair binding energies, $E_n^{(e)} + t_{n,n}^{(e)} + E_{\bar{n}}^{(h)} + t_{\bar{n},\bar{n}}^{(h)} - \epsilon_n$, and behave like \sqrt{B} at large B . An apparent change of slope at $B \approx 3$ Teslas could be an artifact of our finite basis.

In conclusion, we presented a theoretical study of a quantum dot-microcavity system in an homogeneous magnetic field at zero and finite temperatures. We showed that the magnetic field can be used as a control parameter to vary the mean number of photons in the cavity and the critical temperature for BE condensation. The scaling of the total energy and critical temperature with N_{pol} indicates that the effective quasi-particles in the system are weakly interacting quasi-bosons.

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